

Lateral Stability of Precast Prestressed Concrete Girders

In recent years with the use of high strength concrete, 0.6" diameter prestressing strands, and the construction industry's ability to haul and erect increasingly heavy loads it is practical and economical to construct very long precast prestressed concrete girders (> 150 feet). With this capability to design, produce and ship long girders the analysis of lateral girder stability during the design process has become essential. In the past this analysis has often been left to the responsibility of the girder fabricator and hauler. And while it is still ITD policy to require the girder hauler to analyze the girder in transit, the designer should check the girders for stability while lifting them out of the forms and handling them in the yard.

There are two aspects of lateral stability that should be addressed by the designer:

- 1) **Elastic Stability** -- The tendency of the girder to right itself under lateral load.
- 2) **Top Flange Cracking** -- The potential for the girder to fail after initial cracking of the extreme fibers of the top flange due to the lateral bending moments induced by the eccentricity of any initial sweep in the girder.

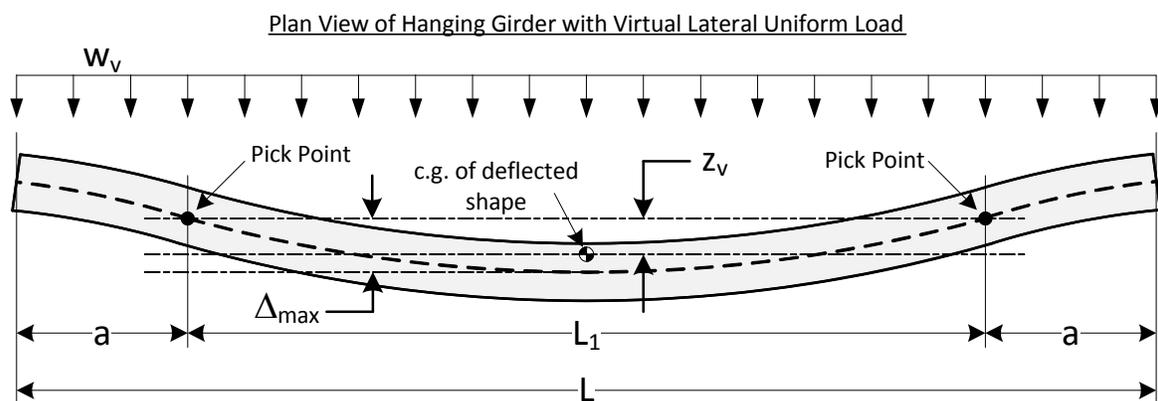
These two phenomena should be treated separately and each one should have an acceptable factor of safety of at least 2.0. It is only necessary to consider the stability while hanging from the pick points, a girder sitting on the bottom flange is inherently more stable because any lateral rotation has to occur about the edge of the bottom flange.

This analysis is very similar to the method presented in the PCI Design Manual, Section 8.10.1.1 Hanging Beams.

Elastic Stability

If it is assumed that a girder is constructed perfectly straight and is suspended from picking points at the center top surface of the girder and inboard from the ends of the girder the elastic lateral stability of the girder in regard to remaining upright can be determined as follows:

Calculate the lateral eccentricity of the center of gravity of the deflected shape of a girder supported inboard from the ends and loaded with a lateral uniform virtual load. With this eccentricity calculate the amount the girder will roll or be canted from vertical. Then determine the lateral component of the uniform self-weight load on the canted girder. As long as the lateral load component of the girder self-weight is less than the assumed virtual load the girder will tend to right itself. Therefore by dividing the virtual uniform load by the lateral self-weight load a factor of safety due to elastic stability can be determined.



L = total length of girder (ft)

a = distance between the end of the girder and the pick point (ft)

L_1 = distance between pick points (ft)

w_{sw} = self-weight uniform vertical load on the girder (kip/ft)

w_v = virtual uniform lateral load on the girder (kip/ft)

E_{ci} = initial concrete modulus of elasticity (ksf)

I_{lat} = moment of inertia about vertical axis which is typically the weak axis of the girder (ft⁴)

z_v = the offset of the center of gravity of the deflected shape due to the virtual load as measured from a line through the neutral axis at the points of support (ft)

$$z_v = K \cdot w_v \quad \text{where:}$$

$$K = \left(\frac{1}{12 \cdot E_c \cdot I_{lat} \cdot L} \right) \cdot (0.1 \cdot L_1^5 - a^2 \cdot L_1^3 + 3 \cdot a^4 \cdot L_1 + 1.2 \cdot a^5)$$

equation for calculating offset of girder c.g. based on the deflected shape of a uniformly loaded beam supported inboard from the ends (ft²/kip)

y_r = the vertical distance from the top surface of the flange at the pick points to the c.g. of the hanging girder (ft)

$$y_r = y_{top} - 0.5 \cdot \Delta_{ic} \quad \text{where:} \quad y_{top} = \text{distance to neutral axis from top of girder (ft)}$$

$$\Delta_{ic} = \text{initial upward camber plus creep of prestressed girder (ft)}$$

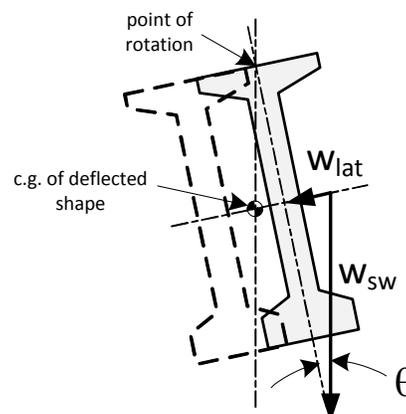
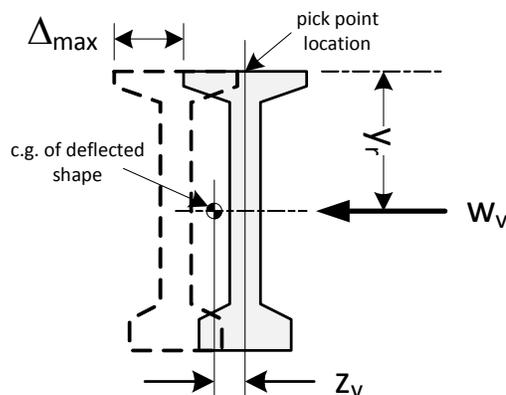
If the hanging girder was flat with no camber the value y_r would equal y_{top} . However since the girder is typically cambered upward due to the combination of prestressing, self-weight and creep, y_r is somewhat reduced from y_{top} . The amount of reduction can be estimated by using one half of the initial camber calculated during girder design, with the creep factors included. It is an approximation to use $0.5\Delta_{ic}$ as the location of the girder c.g. from the neutral axis but it is not critical that this be calculated exactly since the girder camber is relatively small in comparison to y_{top} , (see discussion below on Δ_{sweep} concerning the location of the c.g. of a curved line). The reason for including the creep factors ($1.55\Delta P + 1.65\Delta G$) is that the girder may sit in the yard for months before being picked up and loaded on to trucks, well after the creep effects have occurred.

θ = canted angle of the girder from being deflected laterally from the virtual load (rad)

$$\theta = \text{atan} \left(\frac{z_v}{y_r} \right) = \frac{z_v}{y_r} \quad \text{for small angles } \tan(\theta) \text{ is approximately equal to the angle in radians}$$

w_{lat} = the component of the self-weight of the girder acting laterally on the girder in the canted position (kip/ft)

$$w_{lat} = \sin(\theta) \cdot w_{sw} = \theta \cdot w_{sw} \quad \text{for small angles } \sin(\theta) \text{ is approximately equal to the angle in radians}$$



FS_{elastic} = factor of safety due to the elastic stability of the girder

$$FS_{elastic} = \frac{w_v}{w_{lat}} = \frac{w_v}{(\theta \cdot w_{sw})} = \frac{w_v}{\left(\frac{z_v}{y_r} \cdot w_{sw}\right)} = \frac{w_v}{\left(\frac{w_v \cdot K}{y_r} \cdot w_{sw}\right)} = \frac{1}{\left(\frac{K \cdot w_{sw}}{y_r}\right)} = \frac{y_r}{(K \cdot w_{sw})}$$

z₀ = the offset of the center of gravity of the deflected shape due to a lateral load equal to the self-weight of the girder as measured from a line through the neutral axis at the points of support (ft)

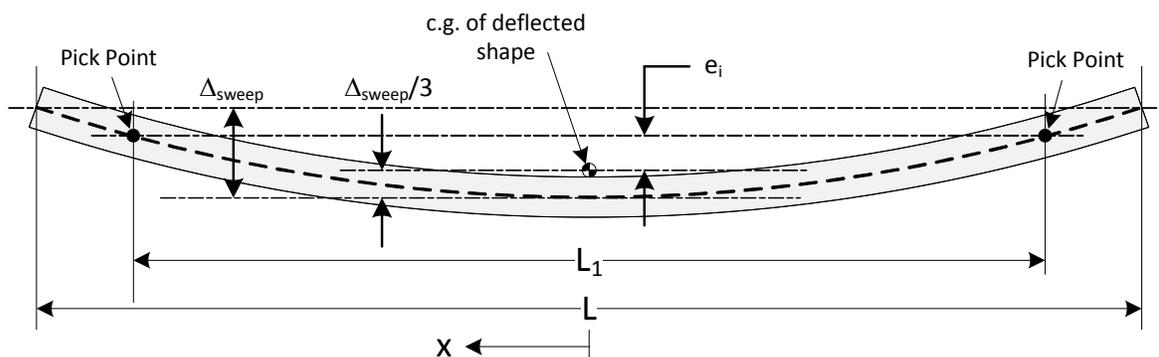
$$z_0 = K \cdot w_{sw} = \left(\frac{w_{sw}}{12 \cdot E_c \cdot I_{lat} \cdot L}\right) \cdot (0.1 \cdot L_1^5 - a^2 \cdot L_1^3 + 3 \cdot a^4 \cdot L_1 + 1.2 \cdot a^5)$$

FS_{elastic} = $\frac{y_r}{z_0}$ The net result simplifies to just y_r divided by z₀, all other terms cancel out. The factor of safety should be greater than 2.

Top Flange Cracking

For the elastic stability calculated above to be maintained it is essential that the concrete section remain uncracked. Cracking in the outer girder flanges reduces the lateral moment of inertia which reduces the girders ability to resist lateral loads. Since all girders have some initial sweep there is an eccentricity to the self-weight which induces lateral load on the girder. If this lateral load on the girder becomes significant the edge of the top flange may crack thereby reducing the effective moment of inertia which will increase lateral deflection increasing lateral load and failure will occur rapidly. By specification the girders are allowed to have an initial sweep due to construction tolerances of 1/8" per ten feet of length. If it is assumed that the sweep of the girder follows a circular curve then the c.g. of the curved line is offset from the point of maximum deflection by Δ_{sweep}/3 (Δ_{sweep} being the maximum deflection at mid-girder). If the girder is actually supported at points inboard from the ends, the offset to the c.g. of the girder from a line through the support points can be calculated as shown below. In addition to checking the potential for cracking of the top flange the overall design stress limits for the initial prestress plus self-weight condition should not exceed the allowable limits (0.200 ksi in tension and 0.19 sqrt f_{ci} in compression)

Plan View of Hanging Girder with Initial Sweep



$$\Delta_{sweep} = \frac{1}{12} \cdot \left[0.125 \cdot \text{in} \cdot \left(\frac{L}{10 \cdot \text{ft}} \right) \right] \quad \text{allowable sweep (ft)}$$

In addition to the allowable tolerance for sweep it is ITD policy to add 0.5" for picking loop tolerance, therefore:

$$e_i = \left[\left(\frac{L_1}{L} \right)^2 - \frac{1}{3} \right] \cdot \Delta_{sweep} + \frac{0.5}{12} \quad \text{equation for calculating eccentricity of c.g. based on circular curve plus 1/2" tolerance for picking loops (ft)}$$

$$\theta_i = \operatorname{atan}\left(\frac{e_i}{y_r}\right) = \frac{e_i}{y_r} \quad \text{initial canted angle due to sweep (rad)}$$

$$M_{\text{crack}} = \frac{1}{12} \cdot (f_r + \sigma_{\text{sw}} + \sigma_{\text{ps}}) \cdot S_{\text{lat}} \quad \text{lateral moment required to crack the outside edge of the top flange at the point under question (kip-ft)}$$

$$f_r = 0.24 \cdot \sqrt{f_{ci}} \quad \text{modulus of rupture of the concrete always positive (ksi)} \\ (f_{ci} \text{ is the initial strength of the concrete in ksi)}$$

$$M_{\text{sw}} = \frac{w_{\text{sw}}}{2} \cdot \left(\frac{L_1^2}{4} - a^2 - x^2 \right) \quad \text{moment at point "x" as measured from the center of the girder due to self-weight (kip-ft)}$$

$$\sigma_{\text{sw}} = 12 \cdot \frac{M_{\text{sw}}}{S} \quad \text{self-weight stress at harp point or mid-span, compression is positive, tension negative (ksi)}$$

S = girder section modulus about horizontal axis which is typically the strong axis (in^3)

σ_{ps} = prestress stress at top of section at harp point or mid-span, compression is positive, tension negative (ksi)

S_{lat} = girder section modulus about vertical axis which is typically the weak axis (in^3)

θ_{max} = maximum canted angle, at which point the girder top flange will crack due to self-weight (rad)

$$\theta_{\text{max}} = \operatorname{asin}\left(\frac{M_{\text{crack}}}{M_{\text{sw}}}\right) \quad \theta_{\text{max}} \text{ should be checked at both the harp point and at mid-span and the lesser value should be used to determine } FS_{\text{cracking}} \text{ (rad)}$$

$$FS_{\text{cracking}} = \frac{\theta_{\text{max}}}{\theta_i} \quad \text{factor of safety due to flange cracking should be } \underline{\text{greater than 2}}$$